Suggested Solutions to:
Regular Exam, Spring 2016
Industrial Organization
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Question 1: Persuasive advertising and competition

To the external examiner: The students had not seen this exact model before. But the model is of course based on material that they have seen in the course.

Part (a)

From the question we have that firm i's profit function is given by

$$\pi_i = \left[a_i - c - \sum_{j=1}^{n} q_j \right] q_i. \tag{1}$$

Differentiating firm i's profit function once with respect to the own output yields

$$\frac{\partial \pi_i}{\partial q_i} = -q_i + \left[a_i - c - \sum_{j=1}^{n} q_j \right].$$

Differentiating it a second time yields $\frac{\partial^2 \pi_i}{\partial q_i^2} = -2$ (so π_i is strictly concave in q_i). Moreover, in the question it is said that we should assume that all firms are active. Therefore, firm i's optimal output choice must be characterized by the first-order condition:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Leftrightarrow q_i = a_i - c - \sum_{j=1}^{n} q_j. \tag{2}$$

Note, incidentally, that this implies that firm i's profit, when the first-order condition holds, equals $\pi_i = q_i^2$ (compare (1)).

Now add up the first-order conditions of the n firms:

$$\sum_{j=1}^{n} q_j = \sum_{j=1}^{n} a_j - nc - n \sum_{j=1}^{n} q_j,$$

which equivalently can be written as

$$\sum_{j=1}^{n} q_j = \frac{\sum_{j=1}^{n} a_j - nc}{n+1}. \tag{3}$$

Plugging (3) into (2) yields

$$\begin{aligned} q_i &= a_i - c - \frac{\sum_{j=1}^{n} a_j - nc}{n+1} \\ &= \frac{(n+1)(a_i - c) - \sum_{j=1}^{n} a_j + nc}{n+1} \\ &= \frac{na_i - \sum_{j \neq i} a_j - c}{n+1}. \end{aligned} \tag{4}$$

Firm i's equilibrium output is thus given by (4), which is what we were supposed to show.

For later reference we also note that we can, using the relationship $\pi_i = q_i^2$, write firm i's equilibrium profit as

$$\pi_i = q_i^2 = \left[\frac{na_i - \sum_{j \neq i} a_j - c}{n+1} \right]^2.$$

Part (b)

Remark: These are the solutions to the version of question 1, part (b), that the students were actually asked to solve. There exists another version of this question, which the students were supposed to be given on exam day through the Digital Exam system but which — because of an unfortunate mistake — they did not get. In the version of Q1b that the students were actually asked to solve, there are two issues:

- I claim that the total advertising expenditures are given by $x^* \varphi(x^*)$. But this is nonsense of course. I meant to write $n\varphi(x^*)$.
- The question is also problematic because of issues with the second-order condition: The

first-order condition will not yield a maximum for a firm. This problem arises partly because I have set the intercept, $\overline{a} - c$, equal to zero. But even for larger values of the intercept, the second-order condition will be problematic if the number of firms is large enough.

I discovered these problems after having submitted my exam paper to the exam administration, but still well before the exam day. I then created a new version of the exam paper, with changes in Q1b, and sent it to the administration. Unfortunately, however, the students were not given the new version of the exam paper but the old and incorrect one (I discovered this a few days after exam day). I'm sorry for that. I'm the one who created the problem by first submitting a version that was wrong. The good news is that the issues with Q1b that I describe above might not have mattered very much for most students. There are not many signs and indications that the students noticed the issues (although there are some exceptions). Nevertheless, I and censor decided to grade generously on this question whenever it looked as if the errors might have created unintended difficulties.

Below you can find the solutions that I obtain when simply ignoring the difficulties with the second-order condition (pretending that it is fine).

We have already solved the stage 2 game. From those calculations we know that, at the stage 2 equilibrium, firm i's profit equals

$$\begin{aligned}\pi_i &= q_i^2 = \left[\frac{n\left(\overline{a}+x_i\right) - \sum_{j\neq i}\left(\overline{a}+x_j\right) - c}{n+1}\right]^2 \\ &= \left[\frac{nx_i - \sum_{j\neq i} x_j + \overline{a} - c}{n+1}\right]^2 \\ &= \left[\frac{nx_i - \sum_{j\neq i} x_j}{n+1}\right]^2 = \frac{\left(nx_i - \sum_{j\neq i} x_j\right)^2}{(n+1)^2}.\end{aligned}$$

Thus firm i's overall profit at stage 1 can be written as

$$\Pi_i = \frac{\left(nx_i - \sum_{j\neq i} x_j\right)^2}{(n+1)^2} - kx_i^3.$$

The first-order condition is given by

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{2n\left(nx_i - \sum_{j\neq i} x_j\right)}{(n+1)^2} - 3kx_i^2 = 0.$$

Imposing symmetry yields

$$\frac{2nx}{(n+1)^2} = 3kx^2 \Rightarrow x^* = \frac{2n}{3k(n+1)^2}.$$

The expression $x^*\varphi(x^*)$ thus becomes $x^*\varphi(x^*) = k(x^*)^4 = An^4/(n+1)^8$ (where A is a constant that does not depend on n). This expression clearly goes to zero as n goes to infinity. Moreover, the expression is strictly decreasing in n (for all $n > 1$), which can be verified by differentiation. So it is maximized at $n = 1$.

For completeness: The expression $n\varphi(x^*)$ — which is indeed the total advertising expenditures — becomes $n\varphi(x^*) = nk(x^*)^3 = Bn^4/(n+1)^6$ (where B is a constant that does not depend on n). This expression clearly goes to zero as n goes to infinity. Moreover, by differentiating one can check that the expression is strictly decreasing in n for all $n > 2$, and that it is strictly increasing in n for all $n < 2$. So total advertising expenditures are maximized at $n = 2$.

Part (c)

(i) The reason why it is problematic is that we normally, when doing welfare analysis in microeconomics, equate higher welfare with a higher degree of preference satisfaction; that is, our measure of welfare is the consumer's utility function (or his/her preferences). But persuasive adverting means, according to our assumption, that the utility function itself (so not only the utility *level*) changes due to advertising. In other words, the yardstick with which we want to measure the welfare change does not stay constant. Hence the conceptual problem.[1]

(ii) The other approach was informative advertising. The idea behind that approach is to suppose that the firm's act of advertising provides consumers with information about something that makes them want to consume more of the good. So, for example, this information could concern the availability of the good or the high quality of the good.

(iii) The reason why multiple demand functions can exist in these models is that individual consumers' willingness to pay for the good, and hence their demand, depend on their beliefs about how many other people will buy. In particular, people are more keen on buying if they

[1] To further clarify what is meant by the above explanation, I can add (but the following is not required by the students for full credit): One situation where we would be able to get around the problem would be if the utility comparison yielded the same result regardless of whether we used the pre-advertising or post-advertising preferences of the consumer as our yardstick.

expect many other people to buy. In this environment there can exist one demand function in which beliefs are pessimistic (in the sense that it is believed that, for any given price, few people will buy); given these pessimistic beliefs the incentives to buy are low, which confirms the pessimistic beliefs. For the same preferences and technology there can exist another demand function in which beliefs are optimistic; this leads to strong incentives to buy, which confirms the optimistic beliefs. Hence, the beliefs (pessimistic or optimistic) are self-confirming.

Question 2: Collusion with fluctuating demand

To the external examiner: The students had seen this model before. It was part of a problem set that was discussed in an exercise class. A somewhat simpler version of the model was also discussed in a lecture.

The (c) part of the question has been changed relative to the problem set. But the new (c) part was also discussed in the exercise classes (Problem 11.9: "Can price signal quality?").

Part (a)

- We must investigate under what conditions a typical firm does not want to deviate from the trigger strategy described in the question, given that the other firm follows the trigger strategy.
- To that end, first note that, if following the equilibrium strategy when the state is s, a firm's overall payoff equals

$$\boxed{\tfrac{1}{n}\pi_s^m + \delta V,} \qquad (5)$$

where

$$V \stackrel{\text{def}}{=} \frac{(1-\lambda)\frac{\pi_L^m}{n} + \lambda\frac{\pi_H^m}{n}}{1-\delta} = \frac{(1-\lambda)\pi_L^m + \lambda\pi_H^m}{n\,(1-\delta)}.$$

In words, the firm will in the current period get the fraction $1/n$ of the monopoly profits given state s. In the following periods the state is not yet known, so what enters as the second term of (5) is the fraction $1/n$ of the the stream of expected monopoly profits, discounted to the present period.

- If making the best possible deviation (which is to just undercut the rival's price), the firm can get (almost)

$$\boxed{\pi_s^m + 0,}$$

because from next period onwards the firm gets a zero profit according to the trigger strategy.

- That is, there is no incentive to deviate if

$$\frac{1}{n}\pi_s^m + \delta V \geq \pi_s^m \Leftrightarrow \boxed{\delta V \geq \tfrac{n-1}{n}\pi_s^m.}$$

This condition must hold both for $s = L$ and $s = H$. Because $\pi_H^m > \pi_L^m$, the high-state condition is the most stringent. Therefore the condition holds for both states if and only if it holds for the high state:

$$\underbrace{\delta\frac{(1-\lambda)\pi_L^m + \lambda\pi_H^m}{n\,(1-\delta)}}_{=\delta V} \geq \frac{n-1}{n}\pi_H^m$$

or, equivalently,

$$\delta \geq \frac{(n-1)\pi_H^m}{(n-1+\lambda)\pi_H^m + (1-\lambda)\pi_L^m} \stackrel{\text{def}}{=} \delta_0. \qquad (6)$$

The last inequality is the one that we were asked to derive. The reasoning above (which investigates the incentives to deviate *on* the equilibrium path) shows that this condition is necessary for the trigger strategy to be part of an SPNE. To be able to conclude that the condition also is sufficient, we must consider the incentives to deviate off the equilibrium path — in particular, we must show that it is optimal for a firm to follow the trigger strategy when being in a punishment phase (given that the above condition is satisfied). However, that is indeed, almost trivially, optimal, since the trigger strategy specifies that the firms should revert to the one shot Nash equilibrium ($p = MC$) in case of a deviation, so the firms are by construction of the trigger strategy making best replies in that situation.

Part (b)

One can, as in a standard repeated game, sustain a collusive equilibrium if the firms care sufficiently much about future profits (high enough discount factor δ). However, in this model, the requirement on the discount factor when having a high demand state is more stringent — the firms must be more patient than in the known-demand model for co-operation to be possible. The reason for this is that in the uncertainty model, in a high demand state, demand will be *unusually* high. The demand realization is by assumption independent over time, so the expected profits tomorrow and onwards are the same regardless of today's demand state. This means that when the demand is known to be high today, then the incentive to deviate from the equilibrium is higher than in the standard model, as the "one-period temptation" is unusually high whereas the "long-term reward of not deviating" is the same. The conclusion is that there is a tendency for collusion to break down in a high demand state (hence price war during booms and counter-cyclical prices).

Part (c)

(i) *What is meant by predatory pricing and limit pricing?*

The idea: a (dominant) firm may start a price war in order to get rid of a competitor.

- If competitor is currently in the market: "**predatory pricing**."
- If competitor is a potential entrant: "**limit pricing**."

Ordover and Willig's (1981) definition of predatory pricing: "Predation is a response to a rival that sacrifices part of the profit that could be earned under competitive circumstances, were the rival to remain viable, in order to induce exit and gain consequent additional monopoly profit."

(ii) How did Milgrom and Roberts (in Tirole's simplified version) model limit pricing? Focus on the key model assumptions and explain how the logic of the model works.

- The incumbent firm's cost is either low or a high,
  - But the potential entrant does not know the cost.
  - If the cost were actually low, then the entrant would not be able to compete profitably and therefore be better off not entering.
- The incumbent does not want the other firm to enter.
  - Therefore, the incumbent has an incentive to try to make the entrant believe it is a low-cost firm (regardless of whether this is true or not).
- The incumbent firm may be able to induce those beliefs in the entrant by charging a very low price early on.
  - The potential entrant, observing this price, might then infer that the incumbent must be a low-cost firm.
  - For only a low-cost firm would have an incentive to charge such a low price. This is because of the single-crossing condition that holds (or is assumed to hold) in this model. This condition implies that the cost on the margin of charging a low price is lower for a low-cost firm than for a high-cost firm.